# Dissertation – James Lambert

## Abstract

The UK like many advanced economies subsides advanced manufacturing sector (Broadberry & Leunig, 2013), particularly if the technology associated are considered ‘Green’. The USA passed the Inflation Reduction Action (IRA), (White House, 2022) and future EU investments will be made by The Green Deal Industrial Plan (GDIP), (European Commission, 2023). There are presumed to be multiply benefits to this decision, however this is a market distortion to a true free market equilibrium. The neo-classical approach (Cordato, 1980) states that without market failures the government should not intervene in the market and that it may produce worse outcomes, government failure (Grand, 2009).The government therefore must show market failures exist that it is trying to counteract and that intervening would be an economic and social benefit to the UK. The first is covered in the Five Case Model, (HM Tresury, 2018). To confirm the latter analysts must perform ex-ante modelling of its impacts (Munby, 2023).

This paper will investigate the potential introduction of General Equilibrium Modelling (GEM) to analysis interventions in advance manufacturing sectors. With a specific focus on the automotive industry due to it being the author’s area of work however there is possibility of applicability to other areas.

## Context

### Government Departmental Changes

The Department for International Trade (DIT) was created on 14th July 2016 and became part of the Department of Business and Trade (DBT) as part of a Machinery of Government (MOG) change on 7th February 2023. DIT had a remit to complete trade policy analysis, for this it the independent report: Trade modelling review expert panel, recommended development of Computable General Equilibrium Model (CGEM) capabilities, (Venables, 2022).

This remit for external policy focus has been combined with Department for Business, Energy & Industrial Strategy (BEIS) domestic business policy. Therefore, allowing government to have a join up domestic and international policy objectives to strengthen UK industry. As DBT embeds this join up policy direction a sharing of analytical approach across the two departments will grow. Therefore, the domestic policy analysis approach described later in Impact/Benefit justification could be expanded by considering the approach previously used for external policy analysis.

Discussing with ex-DIT colleagues the modelling capability has been based on the work of (Böhringer, et al., 2003) and (Wing, 2004).

### Policy Area - Market Failures

There is a strong economic rationale for government intervention in this sector due to the presence of several market failures. Some market failures affect the whole sector, while others are specific to a particular stage, such as R&D investment. Current HMG assessment of different market failures is viewed through the breakdown given in Figure 1. This paper will focus on production side of the industry specifically capital investments. The example of market failures below builds on the information provide in the Green Book 2022, Section 4.23[[1]](#footnote-1).

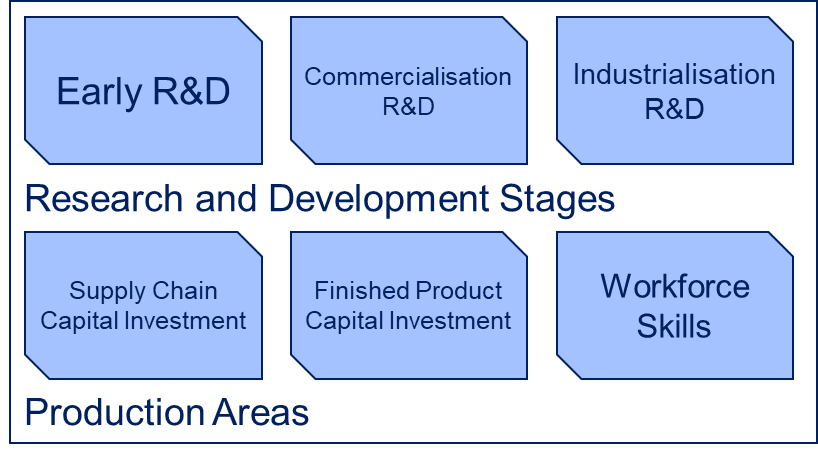


Figure 1 Industry Sections

#### Positive Externality

The industry is current transition to the production of Electric Vehicles (EV) from traditional Internal Combustion Engine (ICEV). EV has a positive externality of reduced CO2 emission when consumed compared to ICEV (USEPA, 2023), even when including the manufacturing process (FCAB, 2021).

As the Marginal Social Benefit (MSB) is greater than the Marginal Private Benefit (MPB) there will be an under consumption and production of the good, (Pindyck & Rubinfeld, 2009).

#### Barriers to Entry and Market Power

For the market to be considered a perfect competition many firms most be present with none having high market power. This is enabled by lower barrier to entry and exit from the market. The automotive industry and its supply chain have high barriers to entry due to the very high initial fixed capital costs; for example, the new gigafactory[[2]](#footnote-2) for JLR will require investment of £4bn (BBC News, 2023).

Certain elements of the electrified supply chain are already characterised by monopolistic supply, for example over 70% of the natural graphite supply is controlled by China.[[3]](#footnote-3)

This market power can enable these Chinese firms to extract profits from European firms by charging above the equilibrium price. The well-established nature of these firms in Asia and the large amount of subsidisation they receive mean that new European firms attempting to find their footing cannot compete.

These results in EV supply chain projects typically having low margins[[4]](#footnote-4) and therefore attract low levels of investable financial especial given the scale required.

#### Information Failures

Although EV have been produced for years, this has not been at the scale required to transfer the entire new car market away from ICEV, a stated goal of the UK government (HMG, 2020) and the EU (Reid, 2023) by 2030 and 2035 respectively. There are multiple competing chemistries that an EV cell can have resulting in uncertainty in investment.

The performance and quality of an EV is driven by this chemistry and the quality of the production process. The knowledge to understand EV cells is complex and therefore the consumers of the good (EV cells) will not be able to have accurate information and the market has failed (Pindyck & Rubinfeld, 2009). This can also be true for car manufactures who use the EV cells as intermediate goods in their production process.

Character of the technology (lock-in) – Vehicle technology is very capital intensive, and so exhibits slow replacement rates. This means that the industry will often keep with existing technologies and be (too) cautious and delay investment until proven alternatives have happened elsewhere before investing significantly into new technologies.

The lock-in nature of the technology means that the following scenario is possible. A cell or car manufacturer has invested in one specific chemistry type which may not develop as productively as other could result in large loss as they are required to retool and invest in the more productive chemistry type.

### Policy Area – Intervention

The above is a summary of the market failures which were used to justify the Automotive Transformation Fund (ATF), (Munby, 2023), as a capital grant fund offer a share of the total valued up to £1 billion, (HMG, 2022) to intervene in the EV supply chain. The total intervention into the sector is called the programme and individual cases are the projects.

#### Impact/Benefit justification

For any grant provided by HMG an ex-ante assessment is completed including an economy appraisal. This is done in line with ‘Green Book’ (GrB) principles. The most common approach is a Cost Benefit Analysis (CBA) appraisal using the Benefit Cost Ratio (BCR) as a metric. The GrB considers the social welfare benefit of bring advanced manufacturing to the UK as the move to more productive jobs. It does not specifical model what the opportunity cost of the economic activity foregone, therefore is a partial equilibrium modelling not general equilibrium.

This approach assumes that the economy is work at or near full employment and therefore only the wage above the next best alternative wage is a benefit. This is assessed against a local alternative.

Advance manufacturing like automotive have higher average wages than alternative employment. The majority of advanced manufacturing occurs outside of London and the Southeast aligning interventionist policies with the levelling up agenda.

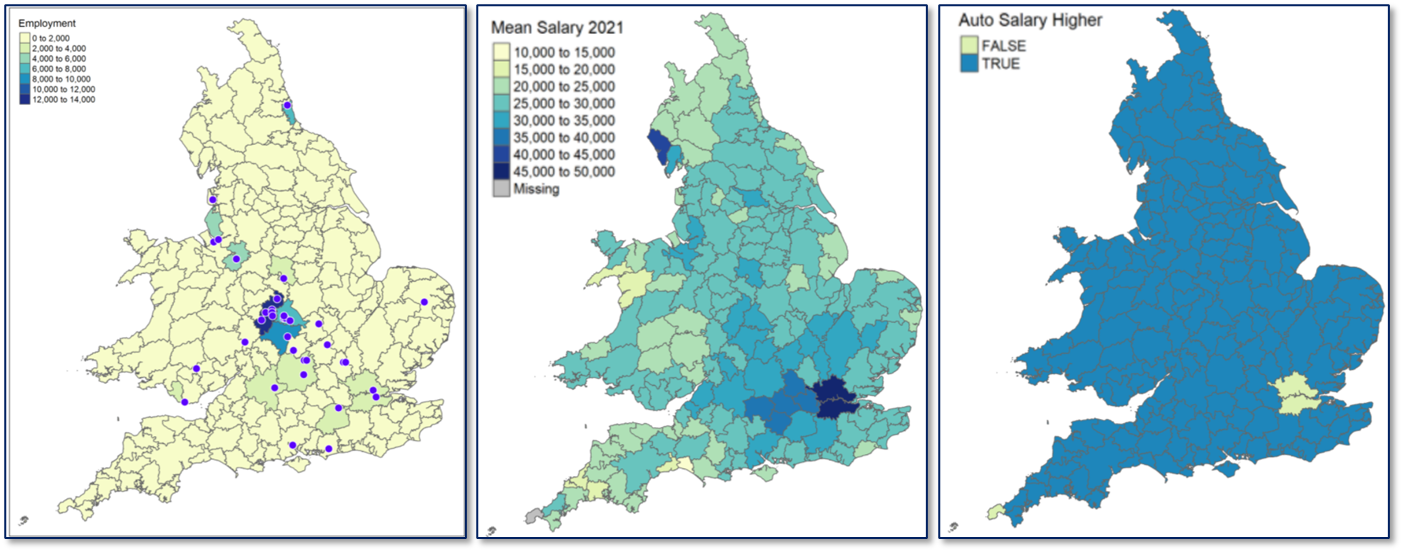


Figure 2 Automotive Employment Maps – ONS data

The maps in Figure 2 show the location of automotive employment, the counterfactual salary, and the areas of the country in which automotive employment represents a move to a more productive employment.

These current assessments are at the project level of individual cases (funding certain OEMs, or key suppliers), but it has been used to approach the programme level intervention. This work will review the macro level of supporting the sector. **This is a live policy debate within HMG and contains commercial sensitive information. Therefore, where appropriate information has been redacted for a wider audience.**

## Methodology

The approach being reviewed is a Computable General Equilibrium Model (CGEM), which is a method of constructing a Walrasian general equilibrium in which supply and demand are equalized across all the interconnected markets in the economy, (Wing, 2004).

There is international precedent for the use of CGEM in policy formation, notably Norway and Australia, (Dixon & Jorgenson, 2013). They have been using single-country models since the 1960’s and 1975 respectively. Since the 1990’s other governments and NGOs have built and maintained CGEM to help policy assessment, including US International Trade Commission, the Organisation for Economic Cooperation and Development, and the World Bank, (Burfisher, 2011).

### Project Aims

To investigate the possibility and usefulness of developing a CGEM to analysis programme level intervention into advance manufacturing sector. The worked example will be the automotive sector and its transition from ICEV to EV production.

### Output Objectives

The output aims for a complete model would be:

* Investigate the structures presented in (Wing, 2004).
* The distribution of employment between different sectors
  + With and without policy intervention
* The comparative salary distribution
* Understand the distribution of wages across areas.
* Produce a framework to analysis regional equality impacts.

Motivation - Objectives

The reasons for these aims are:

* To enable the model to produce a counterfactual to compare to the intervention state to calculate the impact of the policy.
* To enable comparison of modelling results on the same primary economy benefit (wage premium).
* To assess policy impacts against wider strategy of Levelling Up.

The benefit of taking an CGEM approach would be to investigate the opportunity cost of the economic activity foregone. It would consider how the government intervention will impact the different sectors of the economy.

### Economy structure - closed circular economy

A diagram of a company

Description automatically generated

Figure 3 Close circular economy model

The underpinning idea in CGEM is circular flow of commodities in a closed economy. The above model diagram is an autarky version of the model present in the HMRC GEM paper [**reference**].

CGEM can be dynamic and track the evolution of the economy over time in response to a policy change. In the baseline, it is assumed the economy follows a steady-state growth path where all economic activities grow at a constant rate (HM Revenue & Customs, 2013).

### Desired Modelling Assumptions

#### Firms Assumptions

Firms are assumed:

1. To be profit maximises,
2. To be producing homogenous products,
3. To have a homogenous production function within a single sector of the economy,
4. To be producing using labour, capital, and intermittent good as input factors,
5. To be producing with a constant return to scale,
6. To have production modellable with a Cobbs-Douglas (CD) function,
7. The intermediate goods ratio is not re-scale like the over CD function.

#### Households Assumptions

Households are assumed:

1. To be utility maximisers, with a define utility function.
2. To gain income from selling labour as price takers,
3. To save income at a fixed rate
4. To be budget neutral in a single period; income = expenditure + savings

#### Government Assumptions

The government is assumed:

* To have a constant tax and spend policy through out model expect for the policy being assessed.

## Modelling – CGEM

The data on which the CGEM were calculated with is from the ONS – Office for National Statistics. ONS produces Input-Output Analytical Tables which includes product-by-industry section derived from the annual Supply and Use Tables (SUTs) (ONS, 2023). From this the annual use of intermediate goods, labour expenditure and final output can be gathered. This data has disaggregated by SIC code as described in Annex A: Standard Industrial Classification (SIC). This classification of sectors has used at two levels, 39 sectors and 5 sectors.

This data is the appropriate set for this work as it gives the depth of disaggregation (covers a range of sectors), breadth of data points (range of years) and is relative consistent in presentation when matching across datasets.

### Model Derivation

Initial modelling derivation was built from the work presented in the MIT Joint Program on Science and Policy of Global Change report: Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis. (Wing, 2004)

Set of commodities:

Set of industry sectors:

Set of primary factors:

Set of final demands:

For simplicity

It assumes that the circular economy in the model conforms to:

* Cobb-Douglas Economy
* Commodity market clearance
* Factor market clearance
* Full employment
* Zero profit

These assumptions can be formulised as in the equations below:

|  |  |
| --- | --- |
|  | Equation 1 Commodity Market Clearance |
|  | Equation 2 Factor Market Clearance |
|  | Equation 3 Full Employment |
|  | Equation 4 Zero Profit |

Where:

|  |  |
| --- | --- |
| **Variable** | **Definition** |
|  | Total output of commodity |
|  | Intermediate good required for commodity |
|  | Final good use of commodity from factor rent |
|  | Consumption of commodity |
|  | Saving of commodity |
|  | Total available primary factor |
|  | Primary factor used for sector |
|  | Total rent from all primary factors employed |

The households in this model are assumed to be utility maximises (Assumption 8) and therefore the agent problem can be defined as:

|  |  |
| --- | --- |
|  | Equation 5 Agent Utility Maximisation |

The where the function is defined as:

|  |  |
| --- | --- |
|  | Equation 6 Cobb-Douglas Utility Function |

The producers in this are model are assumed to be profit maximises (Assumption 1) and therefore the producer’s problem can be defined as:

Equation 7 Produces Profit Maximisation

Where is specified as the production function. For this model it will be an extension of the CD investigated in Equation 20 to:

Equation 8 Produces Cobb-Douglas with Intermediate Goods

|  |  |
| --- | --- |
| Where: | Equation 9 Constant Return to Scale |

From these equations can be derived as:

The aim is to create a benchmark within which price of each commodity and wage are set to 1. This unity of prices and wages is a modelling assumption to enable algebraic simplification without the loss of generality in the modelling. There is no loss in generality of modelling as the prices and wages are relative to the base year and consumption/output is not modelled as unity. The following are the equations used to calculate the calibration parameters:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

Equation 10 Calibration Equations

With the above definitions for calibration the general equilibrium equations can be reformulated as:

|  |  |
| --- | --- |
|  | Equation 11 Benchmark Commodity Market Clearance |
|  | Equation 12 Benchmark Factor Market Clearance |
|  | Equation 13 Benchmark Full Employment |
|  | Equation 14 Benchmark Zero Profit |

Rearranging Equation 11 to Equation 14 and inserting the calibration definitions from Equation 10, the following deltas can be derived:

|  |  |
| --- | --- |
|  | Equation 15 Delta Consumption |
|  | Equation 16 Delta Primary Factor |
|  | Equation 17 Delta Profit |
|  | Equation 18 Delta Factor Clearance |

This delta function can be combined into a single function of function:

|  |  |
| --- | --- |
|  | Equation 19 Delta Function |

For a given set of calibration inputs a vector is in equilibrium

The difference of from can be interrupted as the amount the vector needs to change to move the model to equilibrium.

### Model 1

For the model to more computable with initial modelling the 5-sector disaggregation has been used. From this 2015 ONS input data was taken and a benchmark CGEM was constructed. This version of the model assumed no savings, capital, imports, exports, or government involvement.

However, this is not the case in the data from ONS which are presented in current price year opposed at a unit production level. So, the outputs had to themselves calibrated to unity price and wages, which is presented in the code: 11\_Model\_1\_Calibration.R.

The ONS data provide a unit input-output for intermediate goods which could be used for the within the model. From this a corresponding set of and . The model constructed its own (technology constant) to allow each sector to create enough output to cover both intermediate goods and final demand .

Implementing this approach, the benchmark model was able to reach general equilibrium as within the realms of computational tolerance, account for floating point errors.

This could be tested with the delta function in Equation 19.

### Model 2

For a more complex version of the model capital will be introduced as an additional required primary factor. The construction of the first began again with the ONS data, with an initial endowment of fixed capital levels in 2015 and employment figures.

The total output was scale to match a salary cost of 1 and capital cost of 0.1 (to represent an economic wide interest rate of 0.1). This model produced a set of outputs include intermediate goods required and final consumption levels for consumers which match the return on labour and capital, . However, as the initial input data is based on real world values, with the additionality complexity of two primary factors the model could not be engineered such that all the deltas were zero.

### Model 2 Next Steps

The next test for the model is varying the initial endogenous variables and so the benchmark model comes to an equilibrium. This requires the creation of an algorithm to search for the equilibrium point. The endogenous variables in Model 2 that could be altered are the vectors of price , wages , and consumption (consumption alters total output ), each of which are containing 5 elements.

This gives 15 variables which could be scaled, and table below present the relationship between the deltas and increase in the variables. When the delta is has a subscript this means the change of a delta for a different commodity.

|  |  |  |  |
| --- | --- | --- | --- |
| **Increase** in Variable |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

The literature recommends an approach similar to a Newton-type steepest-descent optimization algorithm (Kehoe, 1991). An attempt to construct a algorithm to find the equilibrium can be found in the R code: Model\_2\_Calibration.R. However, this was unsuccessful.

## Modelling Weaknesses

Although the CGEM can investigate the opportunity costs of the intervention better than the previous approaches by showing which industries are impacted by incentivising the Automotive sector it does have some weaknesses. This section of the paper will discuss them and give some potential solutions where investigated.

### Benchmark Approach

#### Unity Prices and Wages

The methodology reviewed in the documentation readily used in DBT (ex-DIT) assumes in the benchmark year prices and wages are set to 1. This enables the algebra to construct future years relative to the first year. However, this assumption negates the main benefit that a model would be trying to predict. Therefore, an attempt to construct a version that assumes wages which are not equal to 1 was created. The code can be found in 00\_Main\_Model\_v1.R[[5]](#footnote-5). Code to find the equilibrium point was not produced for this version.

#### Multiplicity of Equilibrium

It is extremely difficult to proof analytically that a given equilibrium solution is unique in real world models, especial when there is the inclusion of taxes distortion. There have been examples testing for multiplicity of equilibria in real-world CGE models. However, the complexity of doing so would means there are uncommon. (Wing, 2004).

This does provide a doubt on the solution and subsequential policy advice provided by any CGE modelling.

#### Production

The methodology presented by (Wing, 2004) and used in the modelling above calculates the CD coefficients for production from a single year of data. Depending on the sensitivity of the results to these coefficients this may lend to misleading results. A potential solution to this is approximate the CD from many years of data.

The three factor Cobb-Douglas is given as:

Where: Output, intermediate goods, capital used, labour used, technology constant.

Equation 20 Factor CD Production Function

The would as be inferred from the ONS Input-Output tables. Linear regression modelling and an author derived data searching algorithms were used to calculate the parameters of Equation 20. Full technical detail of the approach can be found in Annex A: Technical.

The benefit of this approach is to produce more robust coefficients.

#### Utility Function

The CD utility function use in the initial modelling approach does not consider the income-elasticity of demand. Given the modelling is considers increasing the wages of individuals by incentivising more productive employment this missing functionality will not account for the changing demand in the economy.

To improve the model’s utility function, a Constant Difference of Elasticities (CDE) was considered. This which was first introduced by Hanoch (1975) and assumes implicit additivity and enables the incorporation of income effects on the demand system (Hertel, et al., n.d.), (Hertel & van der Mensbrugghe, 2016). A further explanation and results from CDE modelling can be found in Annex A: Technical: 5 Price Elasticity Modelling.

This required the use of LASSO regression methods as the data had multicollinearity, (Altelbany, 2021) because of have joint intermediate goods which would all move with commodity specific inflation.

### Black Box

Simulation models like CGEM are criticized for being ‘Black Boxes’ (Burfisher, 2011), whose results are difficult to explain by clear causal chains (Dixon & Jorgenson, 2013). This is especial true for proprietary commercial modelling, which is the reason is the reason that R an open-source coding language was attempted when creating a new CGEM.

## Conclusion/Summary

This paper was an investigation into the use of CGEM as a policy analysis tool for domestic intervention into an advance manufacturing sector. Due to the weakness discuss above a satisfactory model to appraise the policy decision has not been able to be completed.

Therefore, the ongoing policy decision of intervention level into the advanced manufacturing sectors the previous analytical framework should be continued to be used. However, this does not mean that this approach could not be utilised with additional work. It would require collaboration with analysis with additional skills in programming and data science.

The future model structure recommendation would to be:

* Use of the multi-year Cobb-Douglas Production Function Parameters.
* Intermediate good values from ONS.
* Exogenous constant saving rate.
* Labour - primary factor costs (salary) based on ONS data.
* Incorporating a tax system, simplest being a Poll Tax or flat rate of income and cooperation tax policy.
* Government expenditure would be model as increased consumption of sector goods based on ONS Input-Output table data.

## Annex A: Technical

1. Standard Industrial Classification (SIC)

SIC was first introduced into the UK in 1948 for classifying business establishments (ONS, 2009) and can be used across multiple ONS datasets as a method of managing data disaggregated by production sector.

The classifications are in a hierarchy structure, section (letter), division (2-digit), group (3-digit), class (4-digit) and subclass (5-digit). There are 39 classifications used in parts of this report modelling, which are:

|  |  |  |
| --- | --- | --- |
| **Code** | **Description** | **New Code** |
| A01 | 01: Crop and animal production, hunting and related service activities | A |
| A02 | 02: Forestry and logging | A |
| A03 | 03: Fishing and aquaculture | A |
| B | B: MINING AND QUARRYING | A |
| C10T12 | 10-12: Manufacture of food, drink and tobacco | B |
| C13T15 | 13-15: Manufacture of textiles, wearing apparel, leather and leather products | C |
| C16 | 16: Manufacture of wood and of products of wood and cork | C |
| C17 | 17: Manufacture of paper and paper products | C |
| C18 | 18: Printing and reproduction of recorded media | C |
| C19 | 19: Manufacture of coke and refined petroleum products | C |
| C20 | 20: Manufacture of chemicals and chemical products | C |
| C21 | 21: Manufacture of basic pharmaceutical products and pharmaceutical preparations | C |
| C22 | 22: Manufacture of rubber and plastic products | C |
| C23 | 23: Manufacture of other non-metallic mineral products | C |
| C24 | 24: Manufacture of basic metals | C |
| C25 | 25: Manufacture of fabricated metal products, except machinery and equipment | C |
| C26 | 26: Manufacture of computer, electronic and optical products | C |
| C27 | 27: Manufacture of electrical equipment | C |
| C28 | 28: Manufacture of machinery and equipment | C |
| C29 | 29: Manufacture of motor vehicles, trailers and semi-trailers | D |
| C30 | 30: Manufacture of other transport equipment | C |
| C31\_32 | 31-32: Manufacture of furniture and other manufacturing | C |
| C33 | 33: Repair and installation of machinery and equipment | A |
| D | D: ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY | E |
| E | E: Water supply; sewerage, waste management and remediation activities | E |
| F | F: Construction | E |
| G | G: Wholesale and retail trade; repair of motor vehicles and motorcycles | E |
| H | H: Transportation and storage | E |
| I | I: Accommodation and food service activities | E |
| J | J: Information and communication | E |
| K | K: Financial and insurance activities | E |
| L | L: Real estate activities | E |
| M | M: Professional, scientific and technical activities | E |
| N | N: Administrative and support service activities | E |
| O | O: Public administration and defence; compulsory social security | E |
| P | P: Education | E |
| Q | Q: Human health and social work activities | E |
| R | R: Arts, entertainment, and recreation | E |
| S | S: Other service activities | E |

1. Further Reduction

When completing CGEM the following groupings of industry sectors will be used.

|  |  |
| --- | --- |
| **New Code** | **Definition** |
| **A** | Raw Materials, Manufacturing input |
| **B** | Food goods |
| **C** | Manufactured Non-Food Goods |
| **D** | Automotive Production |
| **E** | Services and Other Goods |

1. Production Function 1

This section gives the mathematical derivation of the linear regression model used to calculate the parameters of the CD production function for each section of the economy. This is an adaption of the methodology for two factor production function found in [**reference**]. This two-factor method has been replicated for SIC 29 to give:

If you assume constant return to scale, then .

Equation 21 Multivariable Linear Regression Model of CD

The constants in this equation can be estimated by multi-variable linear regression which was complete in R. The simple linear regression used to calculate the parameters can return answer in the whole range . Therefore, for some of the sectors there may exist better estimators of the function outside the bounds of the parameters, .

#### Linear Regression Results

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Sector** |  | **Alpha** | **Beta** | **Gamma** |
| A01 | 1.364 | 0.414 | 0.111 | 0.475 |
| A02 | 1.602 | 0.188 | 0.353 | 0.458 |
| B | 1.113 | 0.666 | 0.118 | 0.215 |
| C18 | 1.058 | 0.587 | 0.259 | 0.154 |
| C19 | 1.282 | 0.698 | 0.072 | 0.229 |
| C22 | 0.941 | 0.512 | 0.445 | 0.043 |
| C23 | 1.261 | 0.396 | 0.47 | 0.135 |
| C25 | 1.677 | 0.11 | 0.538 | 0.353 |
| C28 | 1.036 | 0.466 | 0.459 | 0.075 |
| C31\_32 | 0.843 | 0.754 | 0.22 | 0.026 |
| D | 0.873 | 0.752 | 0.087 | 0.162 |
| F | 1.758 | 0.571 | 0.099 | 0.331 |
| H | 1.111 | 0.314 | 0.565 | 0.121 |
| N | 0.952 | 0.597 | 0.384 | 0.02 |
| O | 1.094 | 0.138 | 0.63 | 0.232 |
| P | 1.248 | 0.319 | 0.358 | 0.323 |
| Q | 1.904 | 0.293 | 0.244 | 0.462 |

Table 1 Linear Regression Modelling Results

1. Production Function 2

To find the parameters for the sectors that linear regression failed other techniques were required. The aim was to find the local minimum Mean Squared Error (MSE) within the parameter range. Given the level of accuracy the modelling is going to use, the first approach was to use a brute force search method. Calculate the Mean square Error (MSE) of all possible combination of at 2-digits (i.e., 0.23, 0.76, etc).

This approach would give a local minimum within the discrete search space, that would be a false local minimum of the continuous search space. i.e. it is the minimum MSE for the finite discrete set of parameter searched over.

Where the bar denotes the found estimators satisfying the equation:

,

However, this was too computationally intensive and only one result was found after 18 hours of run time. There were 22 sectors that needed finding (c16 day runtime).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Sector** | **A** | **Alpha** | **Beta** | **Gamma** |
| A03 | 0.95 | 0.8 | 0.01 | 0.199 |

1. Production Function 3

The next search method to find the local minimums was an algorithmic search based on shrinking spheres. The starting point for the algorithm is the central point, .

* The step 1 is to create new set of points are selected at random on the surface of the sphere centred at the start point.
* The radius of the sphere decreases with each iteration of the algorithm.
* The MSE of each element of the set is calculated.
* The element with the smallest MSE is the new start point, return to step 1.

The algorithm is illustrated in Figure 4. The code can be found in LM Regression 7.R, the name of the functions in Figure 4 correspond to those in the code.

A diagram of a function

Description automatically generated

Figure 4 Flow Diagram of Search Algorithm

#### Production Function Results

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Sector** | **A** | **Alpha** | **Beta** | **Gamma** |
| A01 | 3.91 | 0.41 | 0.11 | 0.48 |
| A02 | 4.96 | 0.19 | 0.35 | 0.46 |
| A03 | 29.92 | 0.41 | 0 | 0.58 |
| B | 3.04 | 0.67 | 0.12 | 0.22 |
| C10T12 | 7.23 | 0.7 | 0.03 | 0.27 |
| C13T15 | 8.06 | 0.75 | 0.18 | 0.06 |
| C16 | 3.3 | 0.88 | 0.07 | 0.05 |
| C17 | 141.98 | 0.29 | 0.57 | 0.13 |
| C18 | 2.88 | 0.59 | 0.26 | 0.15 |
| C19 | 3.6 | 0.7 | 0.07 | 0.23 |
| C20 | 4.82 | 0.81 | 0.13 | 0.06 |
| C21 | 84.22 | 0.41 | 0.49 | 0.09 |
| C22 | 2.56 | 0.51 | 0.44 | 0.04 |
| C23 | 3.53 | 0.4 | 0.47 | 0.13 |
| C24 | 3 | 0.87 | 0.07 | 0.06 |
| C25 | 5.35 | 0.11 | 0.54 | 0.35 |
| C26 | 18.37 | 0.62 | 0.31 | 0.08 |
| C27 | 3.57 | 0.87 | 0.08 | 0.06 |
| C28 | 2.82 | 0.47 | 0.46 | 0.07 |
| C29 | 8.08 | 0.73 | 0.2 | 0.06 |
| C30 | 28.15 | 0.58 | 0.4 | 0.02 |
| C31\_32 | 2.32 | 0.75 | 0.22 | 0.03 |
| C33 | 67.14 | 0.4 | 0.43 | 0.16 |
| D | 2.39 | 0.75 | 0.09 | 0.16 |
| E | 6.48 | 0.8 | 0.14 | 0.06 |
| F | 5.8 | 0.57 | 0.1 | 0.33 |
| G | 37.54 | 0.37 | 0 | 0.63 |
| H | 3.04 | 0.31 | 0.56 | 0.12 |
| I | 5.76 | 0.81 | 0.13 | 0.06 |
| J | 16.51 | 0.66 | 0.28 | 0.06 |
| K | 162.35 | 0.32 | 0.62 | 0.07 |
| L | 289.52 | 0.15 | 0.48 | 0.37 |
| M | 60.66 | 0.43 | 0.48 | 0.09 |
| N | 2.59 | 0.6 | 0.38 | 0.02 |
| O | 2.99 | 0.14 | 0.63 | 0.23 |
| P | 3.48 | 0.32 | 0.36 | 0.32 |
| Q | 6.71 | 0.29 | 0.24 | 0.46 |
| R | 17.81 | 0.61 | 0.31 | 0.07 |
| S | 62.86 | 0.41 | 0.48 | 0.12 |

Table 2 Linear Regression and Search Algorithm Results

This coefficients could be used in any future CGEM modelling without the bias of having a single benchmark year.

1. Price Elasticity Modelling

[**to be complete**]

It can be shown that [𝜎𝑖𝑗]𝑁×𝑁 is also symmetric, and the matrix is NSD if and

only if [𝜕𝑞/𝜕𝑝]𝑁×𝑁 is NSD. Therefore, a demand system is regular means 1) the

Slutsky/AUES matrix [𝜎𝑖𝑗]𝑁×𝑁 is NSD; and 2) the Hicksian demand 𝑞 is nonnegative.

For CGE modeling, it is necessary to ensure that the demand system is globally regular (i.e., it should remain regular everywhere in the domain of price).

This is because the algorithm of the solver for finding equilibria may begin from

an initial point of price and quantity combination that is far from the equilibrium levels, and in the process of solving the model, the algorithm might fail if the demand system is not globally regular, even the system is locally regular at the equilibrium points (Perroni and Rutherford, 1998).

<https://www.statology.org/lasso-regression-in-r/>

## Annex B: Code Blocks

Full R code used for this project can be found at:

<https://github.com/James-Lambertcoding/Dissertation>

A qr code on a white background

Description automatically generated

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1. [https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment\_data/file/  
   1063330/Green\_Book\_2022.pdf](https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1063330/Green_Book_2022.pdf) [↑](#footnote-ref-1)
2. Gigafactory is the term for a site making cells to go into the batteries of EVs. [↑](#footnote-ref-2)
3. Redacted internal HMG source. [↑](#footnote-ref-3)
4. Redacted internal HMG source. [↑](#footnote-ref-4)
5. This code runs : 01\_Packages.R & 02\_read\_data\_v2.R & 02a\_Start\_point\_data.R & 03\_Calibration\_v3.R & 04\_CGE\_Functions\_v\_2.R. [↑](#footnote-ref-5)